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AN EXAMINATION OF A DISTRIBUTION OF TAC CONTENDER SOLUTIONS

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National Military Command System Support Center
Washington, D. C.

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AN EXAMINATION OF A DISTRIBUTION OF TAC CONTENDER SOLUTIONS

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It is demonstrated that the TAC CONTENDER air warfare model does not necessarily produce mutually enforceable (optimum) strategies for red and blue; moreover, the bandwidth between the minimax and the maximin can be very large.

NATIONAL MILITARY COMMAND SYSTEM SUPPORT CENTER

Technical Memorandum TM 101-75

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AN EXAMINATION OF A DISTRIBUTION OF TAC CONTENDER SOLUTIONS

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ILLUSTRATIONS

ABSTRACT

The TAC CONTENDER air warfare model has demonstrated that it does not always produce mutually enforceable (optimum) strategies for Red and Blue forces as claimed by the developers of the model; moreover, the bandwidth, which is the "nearness" of the model's game value to the actual game value in terms of net tons of ordnance, can be quite large.

This Technical Memorandum examines these strategies and makes recommendations for certain modifications to make the model more effective.

INTRODUCTION

The research reported on in this technical memorandum was performed i.. response to Blankenship's paper (reference (1)). This research was conducted to verify the results reported by Blankenship, explain the results as a special case of limited importance, or explain the results which might be due to mistaken methods. The last possibility was viewed as being of limited probability but deserving consideration, while the first was viewed as being unlikely due to the confidence built by extensive use of the model and its apparent reliability and intuitively good results. In addition to validating this earlier work, it is believed that the investigation of the distribution of TAC CONTENDER answers could be useful in understanding the "band of enforceability". The final results of this research verify Blankenship's work and amplify its importance.

Before actually stating the problem a cautionary note is presented. Two terms, "adaptive strategies" and "non-adaptive strategies", enter into the problem - with disagreement as to which applies to TAC CONTENDER. Due to their complexity, these terms are defined in the discussion section. As Blankenship notes (and this also applies to the authors' research contained in this memorandum), the tests he conducted only have meaning when TAC CONTENDER is regarded as yielding non-adaptive strategies. Mr. Louis Finch, one of the developers of TAC CONTENDER, contends that it is used properly only when the strategies it yields are regarded as <u>adaptive</u> strategies. Since the various organizations employing the TAC CONTENDER model interpret the strategies generated as being non-adaptive as presented b Falk (reference (2)), this research indicates a need for modifying the use of the model and for further research into the question of what the model does. This research is currently under way.

DISCUSSION

Given the assumption that the TAC CONTENDER strategies under discussion be regarded as non-adaptive the problem is stated below with a general description of TAC CONTENDER and some definitions.

TAC CONTENDER simulates an air war with given inputs, such as numbers of airplanes, resupply of airplanes, shelters, length of the war, sortie rates, etc. It allocates the aircraft available to both sides to four different tasks: combat air support, battlefield defense, airfield attack, and airfield defense. The allocation is made for each of n days of the war, and the set of allocations for each side may be called that side's strategy for the war. In order to associate the value of airpower to a ground war, the model computes the number of tons of ordnance each side delivers in combat air support. The forces for each side are altered daily by modeling aircraft attrited and resupplied throughout the war. TAC CONTENDER purports to compute the "optimum" strategies to approximate the "game value" in terms of the difference of the two sides' tons of ordnance delivered in combat air support. A more complete description of this model may be found in reference (3).

The following definitions are applicable: "Adaptive strategies" refers to considering day by day allocations and adapting the succeeding day's strategy to make best use of the enemy's mistakes of the current day. The implication for a war of n days is that for each possible strategy by one side, there is a counter strategy given for the other side. This is a simple enough concept with certain obvious merits, enough so that one would ask of what use would strategies be which did not take the past into account.

Before attempting to justify non-adaptive strategies, the word "optimum" should be discussed. If there is a decision matrix formed by two sets of decisions, one set for each side, and a pairing of these decisions, requiring each side to decide simultaneously, a pair of decisions can be said to be "optimum". This decision is "optimum" if, given side one's decision, side two has picked the best possible for himself, and given side two's decision, side one has picked the best possible for himself.

The strategies which TAC CONTENDER yields are adaptive in one sense, i.e., each day's allocation depends on the previous day's allocation and the results of that day's fighting - how many planes are left. In another sense, the output of TAC CONTENDER includes only one overall game strategy for each side, which seems to require that the strategies be viewed as non-adaptive. Since an optimum non-adaptive strategy is considered to be better than a set of non-optimal adaptive strategies (Falk (2), p.9), there is justification for non-adaptive strategies being considered.

Since TAC CONTENDER is subject to some misinterpretation by its users, its problems can often be compounded. Obviously, where there is a difference in opinion among the cognoscenti, the users may be forgiven their misinterpretations. We have mentioned the adaptive/non-adaptive question above. Another point which can be misinterpretated is the meaning of the daily strategies which are output. The format is of up to 10 "pure" strategies for each side, with associated "probabilities". The 20 possible "pure" strategies which are often used are given in figure 3. Very often the "probabilities" are interpreted as frequency coefficients in the following sense: Suppose for a particular day, for one side, two pure strategies are listed, numbers 1 and 20, with probabilities 0.5 each. This strategy is interpreted as meaning that 50% of that side's forces for that day should perform airfield defense and 50% battlefield attack. This is not the interpretation which matches with the design of TAC CONTENDER. The interpretation which should be placed on this example is that if the war is played numerous times, and 50% of the games play strategy 1 on that day for that side and 50% play strategy 20 (with similar action for the other side and other days), then the average tonnage difference (net tons) will be as predicted. This interpretation is useless for those wishing to use TAC CONTENDER to produce daily allocations (except for a Monte Carlo model's distribution function input); nevertheless, it is the correct interpretation.

The above discussion provides a general concept of the operation of TAC CONTENDER. The problem arises when TAC CONTENDER is assumed to yield the non-adaptive optimum strategies. Actually there is no claim that the actual optimum has been achieved, but that the TAC CONTENDER game value is "near" the actual game value, in terms of net tons. This "nearness" is referred to as bandwidth. The authors of SABER GRAND (ALPHA) (3) claim TAC CONTENDER maintains a narrow bandwidth. Blankenship (1) showed that in three games the optimum is not achieved and that in at least two of them the bandwidth is not less than 20,000 tons, which in this sense does not appear to be "narrow".

In general, it is believed that Blankenship's findings cast doubt on TAC CONTENDER's ability to perform as advertised. To substantiate this contention, the first task was a check of Blankenship's experimental methods. As expected, no errors of note were discovered. The next step was to produce a distribution of strategies, some playing the TAC CONTENDER Blue strategy against various Red strategies, and some playing the TAC CONTENDER Red strategy against various Blue strategies. This yielded two things, a distribution of results in tons so that standard deviations could be calculated and "nearness" properly evaluated, and a check onthe hypothesis that the TAC CONTENDER result was a "local" optimum rather than a "global" optimum, which would be a reasonable state of affairs. In fact it was found that the TAC CONTENDER result is not a "global" optimum, with no reason to believe it is a "local" optimum, and that the bandwidth is not narrow.

EXPERIMENTAL DESIGN

Figure 1 represents an idealized graph of the Air War Allocation Problem solution space, with both the Red and the Blue strategy sets represented as continuous, one-dimensional variables. This figure assumes the existence of a solution. The axis from left to right is the Red strategy set, the axis extending out from the paper is the Blue strategy set, and the vertical axis is the difference in tons, Blue minus Red, of the game score. With these assumptions, the game solution is represented as being at the saddle point. This is the point for which no greater game score can be obtained by holding the Red strategy constant and varying the Blue strategy and for which no smaller game score can be obtained by holding the Blue strategy constant and varying the Red strategy. In fact the Air War Allocation Problem which is addressed by TAC CONTENDER (with the non-adaptive strategies assumption) has a multi-dimensional solution space, with dimensionality depending on the number of days of the war and the number of pure strategies allowed in the mix. A figure analogous to figure 1 exists, but can't be drawn, as more than 3 dimensions are required.

If the strategy variables of a problem are not continuous, but are discrete and finite, or if only a finite subset of the solution space is known, the information contained in figure 1 can also be represented in tabular form as in figure 2. As before, the saddle point or solution gives the largest game score among the Blue strategies for that Red strategy and the smallest game score among the Red strategies for that Blue strategy. Some of the visual impact of the graph is lost because the ordering of the strategies as they are entered into the table may not correspond with their ordering in the graph if the variables are one-dimensional; or, if the strategy sets are of greater dimension than 1, there is no linear ordering. Whatever the loss of visual impact, there is a compensation in computational ease: only a finite number of points need to be checked. The entry which is simultaneously the smallest in its row and the largest in its column is the saddle point--for the solution set in the table. This last point is very important: if one is dealing with a subset of a solution space, one can only find the optimum strategy pairing and game value for that subset.

The game scores in figure 2 have been selected so that the Blue-Saddle/Red-Saddle strategy pairing gives the saddle point. In all the recent literature on TAC CONTENDER, the point is made that TAC CONTENDER is not exact in its solution, but that it is close. In the example in figure 2, we might suppose that Blue-4/Red-1 is picked as optimal (note it is the saddle point of the restricted table without the saddle entries). The pairing of strategies yields a game score of 1280 rather than the saddle point score of 1295. As a method of checking this pairing without generating the whole table (which would be economically infeasible with a large table), one could generate the row and column which have this pairing as an

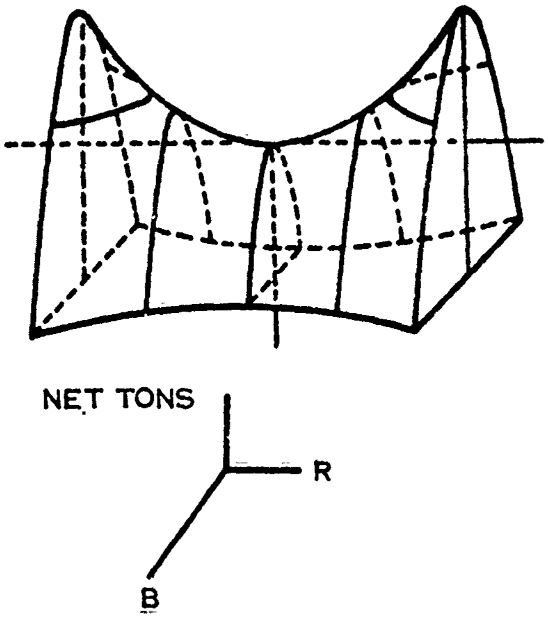

Figure 1. Sample Solution Space

BLUE \ RED	SADDLE	1	2	3	4
SADDLE	1295	1300	1500	1400	1600
1	1000	1200	1405	1280	1500
2	1100	1250	1460	1300	1560
3	900	1000	1100	1050	1200
4	1200	1280	1485	1350	1590

Figure 2. Example Problem

intersection. Checking the row under the assumption that 1280 should be the smallest entry, one finds instead that the smallest is 1200, which is not much smaller. Checking the column under the assumption that 1280 should be the largest entry, one finds that the largest is 1300, which is not much larger. Thus it is concluded that the approximation was close. If the pairing Blue-3/Red-Saddle had been chosen, it can be seen that the approximation is not as close, since there is a larger difference and since Blue-3/Red-Saddle not only is not the largest approximation in its column, but rather it is the smallest.

The TAC CONTENDER problem is similar to the example in the preceding paragraph. TAC CONTENDER presents a game score and daily allocations for each side, which are commonly interpreted as the strategies which are the nearly optimal ones producing the game score near the saddle point, if it exists. The objective is to check to see how close it is to being the smallest entry in its row, how close to being the largest in its column, and the distribution of game scores in each. Since the variables are continuous and multi-dimensional, a complete row or column could not be generated. Instead a sample was taken with an attempt to make the sample representative.

Technical problems were encountered in insuring that the pairs of strategies could be entered into TAC CONTENDER so that the model would evaluate the game as if it had produced them and in generating the variant strategies to be as representative as possible. The first problem was solved by extracting the strategies produced by TAC CONTENDER in the particular game chosen for evaluation, and then inserting these strategies into the modification for the purpose of drawing down the forces of each side and producing the game score. Since the output of the original game and the payoff game agreed, the method was deemed correct. Appendix A is a listing of the file containing the strategies of the original game, with slight format changes for readability. Reading appendix A from left to right, the first number is a strategy number for Blue. (The strategy numbers are defined in figure 3 as to what portion of the force is to be allocated to each of the four tasks.) The second number is a strategy number for Red. The third and fourth numbers are the probabilities for the two strategy numbers for Blue and Red respectively. For each day, there are 10 lines, representing the allowance of up to 10 strategy numbers for that day. As this particular game is a 60-day war, there are 600 strategy-probability lines.

The second problem was solved using a uniform distribution random number generator. Appendix B is a listing of the program which generates the variant Red strategies retaining the TAC CONTENDER Blue strategy. The process involves reading in the 601st line of the strategy file (which is the random number generator seed), generating random numbers, then writing

Strategy Number	Battlefield Attack	Battlefield Defense	Airfield Attack	Airfield Defense
1	0.	0.	0.	1.00
2	0.	0.	.33	.67
3	0.	0.	.67	.33
4	0.	0.	1.00	0.
5	0.	.33	0.	.67
6	0.	.33	.33	.33
7	0.	.33	.67	0.
8	0.	.67	0.	.33
9	0.	.67	.33	0.
10	0.	1.000	0.	0.
11	.33	0.	0.	.67
12	.33	0.	.33	.33
13	.33	0.	.67	0.
14	.33	.33	0.	.33
15	.33	.33	.33	0.
16	.33	.67	0.	0.
17	.67	0.	0.	.33
18	.67	0.	.33	0.
19	.67	.33	0.	0.
20	1.00	0.	0.	0.

Figure 3. Pure Strategies

out the last number as the seed for the next time. Also read in is the 602nd line which tells how many times the file has been used. As each use produces a whole set of daily strategies for Red, and 50 Red variant strategies were produced, this number is incremented from 1 to 50.

The exact process decided upon was to generate a randomly picked strategy number for each non-zero strategy number and to generate new, random frequencies for each strategy number. As some strategy numbers in the original game have zero frequencies (see day 10 of appendix A) thus contributing nothing to that day's strategy, this allows for some days having more strategies in the variant cases than in the original. Further, since the randomizing system allows a strategy number to appear more than once in a day, this means the variant may also have effectively fewer strategies than the original. A constraint is imposed on the frequencies appearing for each side on a given day: they must add to 1.0. To achieve this in the randomized case, while retaining randomness, the random numbers picked as frequencies for a side are summed and each divided by the sum, to normalize them. A similar program was used to generate 50 Blue variant strategies to play against the TAC CONTENDER Red strategy.

These 100 games were played using the TAC CONTENDER payoff modification. The results are tabulated in figure 4. Included also in this table are the figures for the difference in aircraft remaining for each game and the figures for the original game. As can be seen, the figures for the Red variations are larger than those of the original game, as should be for a saddle point. The figures for the Blue variations show that Blue can improve its score, not just by a small amount, but by a large absolute figure, and that the TAC CONTENDER result is in fact almost equal to the mean for the sample. (See figure 7 for means and standard deviations of the various samples.) Thus, in no way is it likely that the strategies of the original game could be near those which produce a saddle point.

A further sample was produced by allowing only strategies 1, 4, 10, or 20 (those strategies with 100% allocations) to Blue, with each third of the war having only one of these allowable strategies, varying over the 64 possibilities and playing these variations against the original Red strategy. This is that half of the sample space tested by Blankenship (1) which produced a contradiction to optimality in this game and as can be seen in figure 5, the results are even more extreme. Figure 6 is a graph of the net aircraft and net tonnage scores derived from figures 4 and 5.

Vary Red Strategies		
Strat. ID	Net Tons	Net A/C
*	1295	-258
Rand# 1	12352	986
Rand# 2	20568	1731
Rand# 3	19097	1422
Rand# 4	14310	1269
Rand# 5	14985	1088
Rand# 6	11534	1334
Rand# 7	6728	993
Rand# 8	13441	1016
Rand# 9	11323	1187
Rand#10	13991	1521
Rand#11	8782	821
Rand#12	8669	819
Rand#13	16739	1455
Rand#14	9432	955
Rand#15	20253	1536
Rand#16	14669	1469
Rand#17	9994	787
Rand#18	20344	1480
Rand#19	21233	1562
Rand#20	11073	1315
Rand#21	17285	1546
Rand#22	13361	1465
Rand#23	11888	1078
Rand#24	[illegible]	1337
Rand#25	[illegible]	[illegible]
Rand#26	[illegible]	[illegible]
Rand#27	13334	1192
Rand#28	22714	1362
Rand#29	[illegible]	[illegible]
Rand#30	[illegible]	[illegible]
Rand#31	13143	[illegible]
Rand#32	[illegible]	[illegible]
Rand#33	17943	[illegible]
Rand#34	[illegible]	[illegible]
Rand#35	[illegible]	[illegible]
Rand#36	[illegible]	[illegible]
Rand#37	22948	[illegible]
Rand#38	10362	[illegible]
Rand#39	23442	1717
Rand#40	[illegible]	1174

Vary Blue Strategies		
Strat. ID	Net Tons	Net A/C
*	1295	-258
Ran1# 1	-2786	-1565
Ran1# 2	1395	-1513
Ran1# 3	-2163	-1474
Ran1# 4	-4736	-1767
Ran1# 5	2845	-1444
Ran1# 6	-377	-1511
Ran1# 7	51	-987
Ran1# 8	3277	-1552
Ran1# 9	3398	-1775
Ran1#10	3225	-1256
Ran1#11	5232	-1315
Ran1#12	-1570	-1592
Ran1#13	4380	-2092
Ran1#14	1374	[illegible]
Ran1#15	-597	-1107
Ran1#16	2682	-1232
Ran1#17	88	[illegible]
Ran1#18	2428	[illegible]
Ran1#19	212	-1266
Ran1#20	[illegible]	-2171
Ran1#21	1492	[illegible]
Ran1#22	[illegible]	[illegible]
Ran1#23	[illegible]	[illegible]
Ran1#24	[illegible]	[illegible]
Ran1#25	[illegible]	[illegible]
Ran1#26	7691	-1775
Ran1#27	[illegible]	-1657
Ran1#28	[illegible]	[illegible]
Ran1#29	[illegible]	[illegible]
Ran1#30	[illegible]	[illegible]
Ran1#31	957	[illegible]
Ran1#32	-2171	[illegible]
Ran1#33	[illegible]	[illegible]
Ran1#34	[illegible]	[illegible]
Ran1#35	[illegible]	[illegible]
Ran1#36	[illegible]	[illegible]
Ran1#37	[illegible]	[illegible]
Ran1#38	1091	[illegible]
Ran1#39	[illegible]	[illegible]
Ran1#40	[illegible]	-1394

* is the original TAC CONTENDER strategy pair

Figure 4. Results of Random Strategy Runs (Part 1 of 2)

Vary Red Strategies			Vary Blue Strategies		
Strat. ID	Net Tons	.Net A/C	Strat. ID	Net Tons	Net A/C
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1236	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1134	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1137	[illegible]

Figure 4. Results of Random Strategy Runs (Part 2 of 2)

Vary Blue Strategies

Strat. ID	Net Tons	Net A/C	Strategy Numbers By Day 1-20	21-40	41-60
*	1295	-258	varied strategies		
Ran2" 1	-3871	-1471	1	1	1
Ran2" 2	-2290	-384	4	1	1
Ran2" 3	-16658	-2668	10	1	1
Ran2" 4	21468	-2672	20	1	1
Ran2" 5	-5181	-1429	1	4	1
Ran2" 6	-2320	-427	4	4	1
Ran2" 7	-16740	-2627	10	4	1
Ran2" 8	21627	-2628	20	4	1
Ran2" 9	-5153	-1552	1	10	1
Ran2"10	-3810	-472	4	10	1
Ran2"11	-16770	-2663	10	10	1
Ran2"12	21508	-2647	20	10	1
Ran2"13	-1136	-1564	1	20	1
Ran2"14	-1122	-497	4	20	1
Ran2"15	[illegible]	-2549	10	20	1
Ran2"16	22772	-2532	20	20	1
Ran2"17	[illegible]	-1440	1	1	4
Ran2"18	-1012	-296	4	1	4
Ran2"19	-16535	-2697	10	1	4
Ran2"20	21579	-2613	20	1	4
Ran2"21	-5160	-1376	1	4	4
Ran2"22	-2847	[illegible]	4	4	4
Ran2"23	-16436	-2568	10	4	4
Ran2"24	21742	-2571	20	4	4
Ran2"25	-5692	-1555	1	10	4
Ran2"26	-3108	-436	4	10	4
Ran2"27	-16660	-2604	10	10	4
Ran2"28	21619	-2599	20	10	4
Ran2"29	-1980	-1517	1	20	4
Ran2"30	-1204	-457	4	20	4

* is the original TAC CONTENDER strategy pair

Figure 5. Results of Blankenship's Selections (Part 1 of 2)

Vary Blue Strategies

Strat. ID	Net Tons	Net A/C	Strategy Numbers By Day 1-20	21-40	41-60
Par2131	-14906	-2491	10	20	4
Par2132	22892	-2482	20	20	4
Par2133	-3621	-1328	1	1	10
Par2134	-1939	-283	4	1	10
Par2135	-16518	-2625	10	1	10
Par2136	21582	-2630	20	1	10
Par2137	-4844	-1338	1	4	10
Par2138	-2631	-363	4	4	10
Par2139	-16422	-2585	10	4	10
Par2140	21759	-2588	20	4	10
Par2141	-4283	-1382	1	10	10
Par2142	-2977	-374	4	10	10
Par2143	-16657	-2621	10	10	10
Par2144	21626	-2607	20	10	10
Par2145	-782	-1468	1	20	10
Par2146	-1312	-430	4	20	10
Par2147	-14995	-2508	10	20	10
Par2148	22890	-2499	20	20	10
Par2149	-2670	-1440	1	1	20
Par2150	-1122	-457	4	1	20
Par2151	-15462	-2595	10	1	20
Par2152	22 72	-2505	20	1	20
Par2153	-4457	-1369	1	4	20
Par2154	-2265	-452	4	4	20
Par2155	-15454	-2466	10	4	20
Par2156	22[illegible]	-2476	20	4	20
Par2157	-4393	-1571	1	10	20
Par2158	-22[illegible]	[illegible]	4	10	20
Par2159	-15[illegible]	-2499	10	10	20
Par2160	22612	-2483	20	10	20
Par2161	-385	-1505	1	20	20
Par2162	-498	-525	4	20	20
Par2163	-13[illegible]	-2327	10	20	20
[illegible]	[illegible]	-2382	20	2	20

Figure 5. (Part 2 of 2)

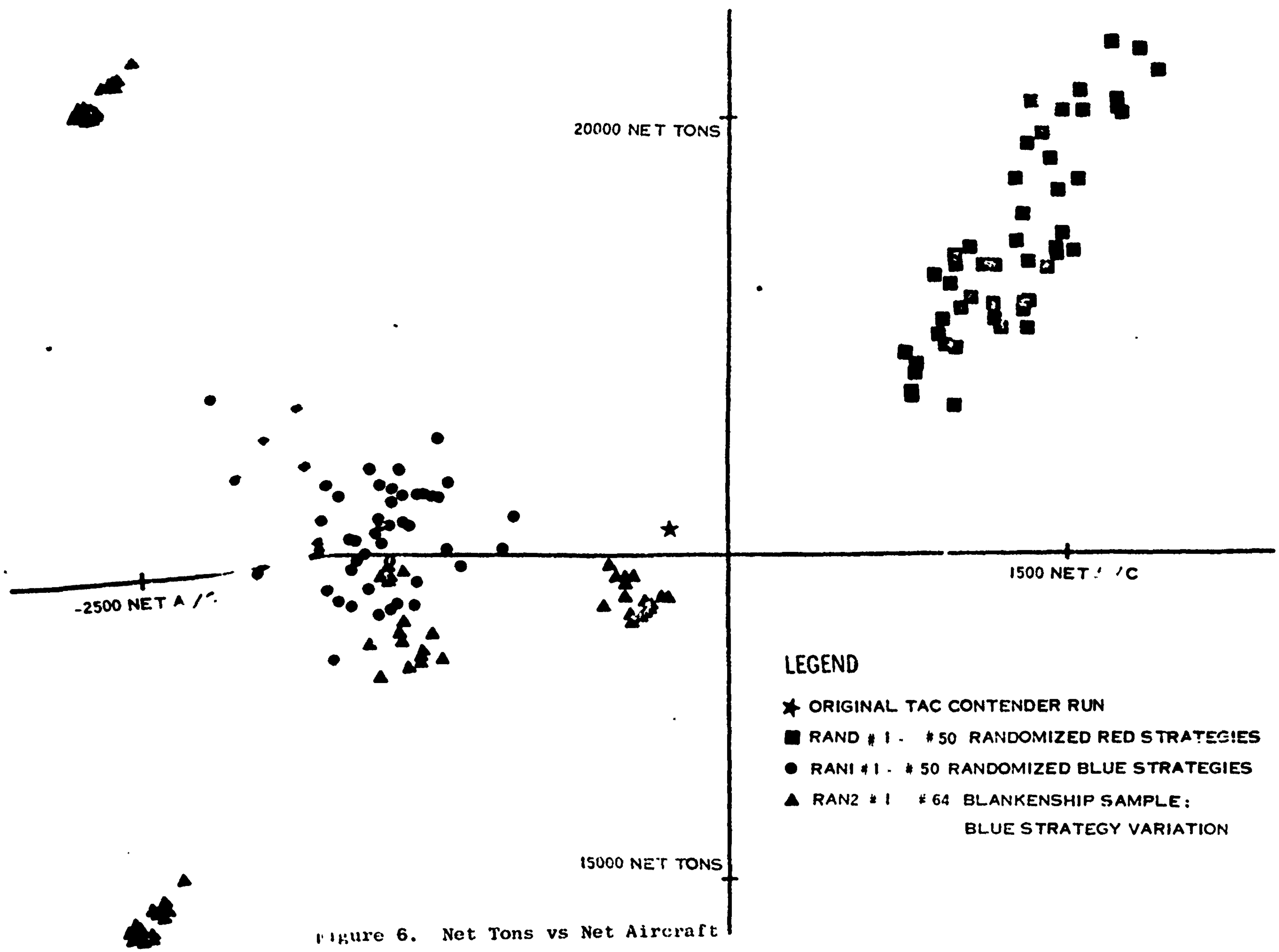

Figure 6. Net Tons vs Net Aircraft

TAC CONTENDER Results

Net Tons	Net Aircraft
1295	-258

RANDOM RED vs TAC CONTENDER BLUE Results

	Net Tons	Net Aircraft
Mean	14225.60	1269.98
Standard Deviation	4557.44	294.20

RANDOM BLUE vs TAC CONTENDER RED Results

	Net Tons	Net Aircraft
Mean	1291.48	-1570.46
Standard Deviation	2642.37	267.10

64 Pure BLUE vs TAC CONTENDER RED Results

	Net Tons	Net Aircraft
Mean	150.53	-1749.89
Standard Deviation	13951.47	897.72

Figure 7. Sample Means and Standard Deviations

RECOMMENDATIONS

Even from this sample, it is obvious that the TAC CONTENDER strategies cannot be regarded as optimal as far as the advertised measure, net tonnage, is concerned. (An interesting fact was noted. It appears that the original game may be a saddle point for the aircraft scores, as is the case for this sample space.) This implies that any past result based on TAC CONTENDER output should be reviewed.

Major modifications to the output of TAC CONTENDER and its interpretation should be made. The output concerning the daily strategies should be suppressed, thus avoiding the temptation to misinterpret the strategies. Essentially only the graph which shows convergence of wars and the payoff table at the end should be retained as output. Further analysis of the functions and utility of the model is recommended.

RE ,RENCES

Footnoted Bibliography

1. Blankenship, Jerry, An Examination of the Width of the Band of Enforceability of TAC CONTENDER Solutions, Working Paper WP-21 Institute for Defense Analyses, TAC NUC, May 1974.

2. Falk, James E., Remarks on Sequential Two Person Zero-Sum Games and TAC CONTENDER, Institute for Management Science and Engineering, The George Washington University, March 1973.

3. Methodology for Use in Measuring the Effectiveness of General Purpose Forces, Saber Grand (Alpha), United States Air Force, Assistant Chief of Staff, Studies and Analysis, March 1971.

Additional Bibliography

1. Anderson, Lowell Bruce, A Brief Review of Some Air-to-Air Models, Working Paper WP-29 Institute for Defense Analyses, Improved Methodologies for General Purpose Forces Planning (New Method Study) August 1972.

2. Anderson, Lowell Bruce, Some Comments on TAC CONTENDER and WP-1, Working Paper WP-33 Institute for Defense Analyses, Improved Methodologies for General Purpose Forces Planning (New Method Study), December 1972.

3. Anderson, Lowell Bruce and Karr, Alan F. Another Type of Counterexample to TAC CONTENDER, Working Paper WP-64, Institute for Defense Analyses, Improved Methodologies for General Purpose Forces Planning (New Methods Study), March 1973.

4. Blackwell, David and Girshick, M.A., Theory of Games and Statistical Decisions, John Wiley and Sons, Inc., New York 1954.

5. Bracken, Jerome and McGill, James T., Review of TAC CONTENDER, Working Paper WP-1, Institute for Defense Analyses, Improved Methodologies for General Purpose Forces Planning (New Method Study), October 1971.

APPENDIX A

Original Game Strategies

DAY	STRATEGIES BLUE	RED	PROBABILITIES BLUE	RED
1	20	9	0.	0.
	2	3	0.	0.1000000000E 01
	3	0	0.1000000000E 01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
2	3	3	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
3	3	3	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
4	3	2	0.9680000000E 00	0.5600000000E 00
	1	4	0.3200000000E-01	0.
	0	3	0.	0.4400000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. Original Game Strategies
(Part 1 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
5	3	3	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
6	3	3	0.	0.1000000000E 01
	2	0	0.1000000000E 01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
7	2	3	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
8	2	3	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 2 of 15)

DAY	STRATEGIES BLUE	RED	PROBABILITIES BLUE	RED
9	2	11	0.1000000000E 01	0.
	10	3	0.	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
10	2	11	0.9120000000E 00	0.
	10	2	0.	0.1120000000E 00
	1	3	0.1600000000E-01	0.8800000000E 00
	11	7	0.7200000000E-01	0.8000000000E-02
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
11	2	2	0.7720000000E 00	0.2800000000E-01
	1	3	0.2000000000E-01	0.9160000000E 00
	11	7	0.1800000000E 00	0.8000000000E-02
	6	11	0.	0.4800000000E-01
	7	0	0.2800000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
12	2	2	0.7280000000E 00	0.8000000000E-01
	1	3	0.1600000000E-01	0.8760000000E 00
	11	7	0.2480000000E 00	0.2800000000E-01
	7	11	0.8000000000E-02	0.1600000000E-01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 3 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
13	2	2	0.7280000000E 00	0.9200000000E-01
	11	7	0.2320000000E 00	0.4800000000E-01
	9	11	0.8000000000E-02	0.
	5	3	0.	0.8600000000E 00
	1	0	0.3200000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
14	2	2	0.5560000000E 00	0.3000000000E 00
	11	7	0.2280000000E 00	0.1440000000E 00
	9	11	0.	0.
	10	3	0.	0.5[illegible] 00
	5	0	0.	0.
	1	0	0.9200000000E-01	0.
	7	0	0.4000000000E-01	0.
	3	0	0.8400000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
15	2	2	0.8000000000E-02	0.3880000000E 00
	11	9	0.1200000000E 00	0.
	1	7	0.2120000000E 00	0.1840000000E 00
	9	11	0.8000000000E-02	0.
	7	3	0.6400000000E-01	0.4280000032E 00
	3	0	0.5880000000E 00	0.
	17	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
16	3	2	0.5120000000E 00	0.4560000000E 00
	11	9	0.	0.
	1	7	0.2720000000E 00	0.2360000000E 00
	2	11	0.8000000000E-02	0.
	9	3	0.	0.3079999968E 00
	7	0	0.8000000000E-01	0.
	19	0	0.1200000000E-01	0.
	17	0	0.1160000000E 00	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 4 of 15)

DAY	STRATEGIES BLUE	RED	PROBABILITIES BLUE	RED
17	3	2	0.3079999968E 00	0.5040000000E 00
	11	9	0.	0.
	1	7	0.3560000032E 00	0.3120000000E 00
	2	11	0.8000000000E-02	0.
	9	3	0.	0.1840000000E 00
	7	0	0.2200000000E 00	0.
	19	0	0.1200000000E-01	0.
	17	0	0.9600000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
18	1	4	0.4240000000E 00	0.
	18	9	0.	0.2000000000E-01
	2	2	0.8000000000E-02	0.5160000000E 00
	17	7	0.9999999999E-01	0.3120000000E 00
	7	11	0.4520000000E 00	0.
	19	3	0.1600000000E-01	0.1520000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
19	7	2	0.4360000000E 00	0.5400000000E 00
	17	9	0.1080000000E 00	0.3200000000E-01
	1	7	0.4280000000E 00	0.3240000000E 00
	2	11	0.8000000000E-02	0.
	5	3	0.	0.1040000000E 00
	14	0	0.	0.
	19	0	0.2000000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
20	7	2	0.4080000000E 00	0.5640000000E 00
	17	9	0.9999999999E-01	0.4000000000E-01
	1	7	0.4640000000E 00	0.3520000032E 00
	2	11	0.8000000000E-02	0.
	5	3	0.8000000000E-02	0.4400000000E-01
	14	6	0.1200000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 5 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
21	1	3	0.4880000000E 00	0.2400000000E-01
	20	9	0.	0.4800000000E-01
	7	2	0.3759999968E 00	0.5680000064E 00
	2	11	0.	0.
	17	7	0.1040000000E 00	0.3400000000E 00
	10	6	0.1200000000E-01	0.2000000000E-01
	14	0	0.2000000000E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
22	1	3	0.5400000000E 00	0.5200000000E-01
	20	9	0.	0.1080000000E 00
	7	2	0.3440000000E 00	0.5400000000E 00
	17	7	0.9999999999E-01	0.2800000000E 00
	2	6	0.8000000000E-02	0.2000000000E-01
	3	0	0.	0.
	14	0	0.8000000000E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
23	1	3	0.5320000000E 00	0.2800000000E-01
	20	9	0.	0.1200000[illegible] 00
	7	2	0.3360000000E 00	0.5840000000E 00
	17	7	0.1240000000E 00	0.2680000000E 00
	3	0	0.8000000000E-02	0.
	2	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
24	1	3	0.5289855104E 00	0.1449275360E-01
	20	9	0.	0.1449275370E 00
	7	2	0.3333333312E 00	0.5652173888E 00
	17	7	0.1304347840E 00	0.2753623200E 00
	2	0	0.7246376832E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 6 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
25	1	17	0.5273972608E 00	0.
	16	9	0.	0.1232876720E 00
	9	6	0.	0.3424657568E-01
	17	2	0.1027397264E 00	0.5547945216E 00
	7	7	0.3013698656E 00	0.2876712352E 00
	14	0	0.6849315072E-02	0.
	5	0	0.6164383552E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
26	1	17	0.2290748896E 00	0.
	16	9	0.	0.1541850208E 00
	9	6	0.	0.1321585904E-01
	7	2	0.2158590304E 00	0.5594713664E 00
	17	7	0.2643171808E-01	0.2731277536E 00
	14	0	0.	0.
	5	0	0.4889867840E 00	0.
	20	0	0.3964757696E-01	0.
	0	0	0.	0.
	0	0	0.	0.
27	5	3	0.5522388096E 00	0.
	20	9	0.6467661696E-01	0.2487562176E-01
	7	2	0.2139303472E 00	0.4179104480E 00
	17	6	0.9950248704E-02	0.3482587072E 00
	1	7	0.1393034832E 00	0.2089552240E 00
	14	12	0.1990049744E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
28	5	3	0.5546218432E 00	0.
	20	9	0.6722689024E-01	0.2941176480E-01
	7	2	0.2100840336E 00	0.4285714304E 00
	17	6	0.	0.3067226880E 00
	1	12	0.1092436976E 00	0.8403361280E-02
	14	7	0.2100840320E-01	0.2268907552E 00
	10	0	0.3781512608E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 7 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
29	5	12	0.6187845312E 00	0.1104972368E-01
	19	9	0.	0.1657458560E-01
	9	6	0.	0.3591160224E 00
	20	2	0.4972375680E-01	0.4088397824E 00
	1	7	0.9392265216E-01	0.2044198912E 00
	7	0	0.1491712704E 00	0.
	14	0	0.2209944736E-01	0.
	10	0	0.6629834240E-01	0.
	0	0	0.	0.
	0	0	0.	0.
30	5	12	0.4129353248E 00	0.9950248704E-02
	19	9	0.	0.1492537312E-01
	9	6	0.	0.4129353248E 00
	20	2	0.1990049744E-01	0.4079601984E 00
	1	7	0.6467661696E-01	0.1542288544E 00
	7	0	0.6965174144E-01	0.
	14	0	0.1492537312E-01	0.
	10	0	0.4179104480E 00	0.
	0	0	0.	0.
	0	0	0.	0.
31	10	2	0.6363636352E 00	0.3681818176E 00
	20	9	0.1818181808E-01	0.1409090912E 00
	7	6	0.5909090880E-01	0.3772727264E 00
	1	12	0.2863636352E 00	0.
	16	7	0.	0.1136363632E 00
	19	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
32	10	2	0.6851063808E 00	0.3914893632E 00
	20	9	0.1276595744E-01	0.1914893616E 00
	7	6	0.4255319168E-01	0.3659574464E 00
	1	12	0.2510638304E 00	0.
	16	3	0.8510638272E-02	0.5106382976E-01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 8 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
33	10	2	0.9481481472E 00	0.1481481472E 00
	20	9	0.1481481472E-01	0.2222222240E-01
	7	6	0.2962962944E-01	0.8296296320E 00
	16	0	0.7407407424E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
34	10	2	0.9520547968E 00	0.1917808224E 00
	20	9	0.1369863008E-01	0.2054794528E-01
	7	6	0.2739726016E-01	0.7876712320E 00
	16	0	0.6849315072E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
35	10	2	0.9440000000E 00	0.5600000000E 00
	20	9	0.1200000000E-01	0.2120000000E 00
	7	6	0.4400000000E-01	0.2230000000E 00
	16	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
36	10	2	0.9568965504E 00	0.4310344832E-01
	20	9	0.8620689664E-02	0.2586206880E-01
	7	6	0.2586206880E-01	0.9310344832E 00
	16	0	0.8620689664E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 9 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
37	10	2	0.9603174656E 00	0.3968253952E-01
	20	9	0.7936507904E-02	0.2380952384E-01
	7	6	0.2380952384E-01	0.9365079296E 00
	16	0	0.7936507904E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
38	10	2	0.9632353024E 00	0.3676470592E-01
	20	9	0.7352941184E-02	0.2205882368E-01
	7	6	0.2205882368E-01	0.9411764736E 00
	16	0	0.7352941184E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
39	10	2	0.9662162176E 00	0.3378378368E-01
	20	9	0.6756756736E-02	0.2027027024E-01
	7	6	0.2027027024E-01	0.9459459456E 00
	16	0	0.6756756736E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
40	10	2	0.9687500032E 00	0.3125000000E-01
	20	9	0.6250000000E-02	0.1875000000E-01
	7	6	0.1875000000E-01	0.9500000000E 00
	16	0	0.6250000000E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 10 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
41	10	2	0.9759036160E 00	0.3012048192E-01
	20	9	0.6024096384E-02	0.1807228912E-01
	7	6	0.1204819280E-01	0.9518072320E 00
	16	0	0.6024096384E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
42	10	2	0.9776536320E 00	0.2793296090E-01
	20	9	0.5586592192E-02	0.1675977048E-01
	7	6	0.1117318432E-01	0.9553072640E 00
	16	0	0.5586592192E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
43	10	2	0.9791666688E 00	0.2604166688E-01
	20	9	0.5208333312E-02	0.1562500000E-01
	7	6	0.1041666664E-01	0.9583333376E 00
	16	0	0.5208333312E-02	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
44	10	2	0.9826839808E 00	0.2164502176E-01
	20	9	0.	0.3896103904E-01
	7	6	0.1731601728E-01	0.9393939328E 00
	19	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 11 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
45	10	2	0.9861111168E 00	0.2314814816E-01
	20	9	0.	0.4166666656E-01
	7	6	0.1388888896E-01	0.9351851904E 00
	19	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
46	10	20	0.9720000000E 00	0.
	19	9	0.8000000000E-02	0.3200000000E-01
	9	6	0.	0.9200000000E 00
	7	2	0.2000000000E-01	0.4800000000E-01
	20	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
47	10	20	0.9350649344E 00	0.3376623392E 00
	19	9	0.3896103904E-01	0.1212121216E 00
	9	6	0.2597402592E-01	0.3722943712E 00
	7	11	0.	0.
	0	17	0.	0.1688311696E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
48	10	20	0.9320000000E 00	0.1520000000E 00
	19	9	0.3600000000E-01	0.3400000000E 00
	9	6	0.1200000000E-01	0.8400000000E-01
	7	2	0.2000000000E-01	0.2000000000E-01
	20	17	0.	0.4040000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 12 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
49	10	20	0.9000000000E 00	0.4400000000E-01
	19	9	0.5600000000E-01	0.4400000000E 00
	9	6	0.	0.6000000000E-01
	7	2	0.4400000000E-01	0.2000000000E-01
	20	17	0.	0.4360000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
50	10	20	0.7400000000E 00	0.1160000000E 00
	20	9	0.	0.3960000032E 00
	9	6	0.3600000000E-01	0.1200000000E 00
	19	2	0.1080000000E 00	0.2000000000E-01
	7	17	0.1160000000E 00	0.3480000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
51	10	20	0.8400000064E 00	0.3200000000E-01
	20	9	0.	0.4800000000E 00
	9	6	0.	0.5600000000E-01
	19	2	0.1040000000E 00	0.8000000000E-02
	7	17	0.5600000000E-01	0.4240000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
52	7	20	0.8400000064E 00	0.2200000000E 00
	20	9	0.	0.4800000000E 00
	15	6	0.6000000000E-01	0.2400000000E-01
	9	17	0.	0.2760000000E 00
	19	0	0.9999999999E-01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 13 of 15)

DAY	STRATEGIES BLUE	STRATEGIES RED	PROBABILITIES BLUE	PROBABILITIES RED
53	7	11	0.4920000000E 00	0.
	20	16	0.	0.
	19	9	0.2920000000E 00	0.4240000032E 00
	10	20	0.2160000000E 00	0.
	15	6	0.	0.2840000032E 00
	9	17	0.	0.2920000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
54	7	11	0.1680000000E 00	0.
	20	16	0.7200000000E-01	0.
	19	9	0.2640000000E 00	0.2120000000E 00
	10	20	0.1720000000E 00	0.
	9	17	0.3240000000E 00	0.1400000000E 00
	15	6	0.	0.3200000000E 00
	0	15	0.	0.3280000032E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
55	9	17	0.4800000000E 00	0.8000000000E-01
	20	16	0.	0.8000000000E-02
	19	9	0.4400000000E 00	0.2600000032E 00
	10	6	0.	0.3160000000E 00
	7	15	0.8000000000E-01	0.3360000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
56	9	17	0.4720000000E 00	0.7600000064E-01
	20	16	0.	0.2400000000E 00
	19	15	0.5280000000E 00	0.6840000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

Figure 8. (Part 14 of 15)

DAY	STRATEGIES BLUE	RED	PROBABILITIES BLUE	RED
57	19	16	0.5280000000E 00	0.8280000000E 00
	9	17	0.4720000000E 00	0.1720000000E 00
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
58	19	16	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
59	19	16	0.1000000000E 01	0.1000000000E 01
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
60	19	20	0.	0.1000000000E 01
	20	0	0.1000000000E 01	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.
	0	0	0.	0.

SEED 0.0234567890E 00
COUNTER 1

Figure 8. (Part 15 of 15)

APPENDIX B

Red Strategy Program Listing

```
C*
C*      USERAND RANDOMIZES THE NON-ZERO SIDE 2 STRATEGIES
C*      IN FILE 12 AND GIVES RANDOM FREQUENCIES FOR THEIR
C*      PLAY.
        DIMENSION IIVEC(60,10,2),TFREQ(60,10,2)
C       READ FILE 12
        DO 1 IIN=1,60
        DO 1 JJN=1,10
        READ(12,2) ((IIVEC(IIN,JJN,KKN),KKN=1,2),
       &(TFREQ(IIN,JJN,KKN),KKN=1,2))
      1 CONTINUE
      2 FORMAT(2I10,2E17.10)
        READ(12,3) SEED
      3 FORMAT(E17.10)
        READ(12,4) JSEED
      4 FORMAT(I10)
        WRITE(6,5) JSEED
      5 FORMAT(' THIS IS THE RESULT OF RANDOMIZING',
       &' THE ORIGINAL STRATEGIES ',I10,' TIMES.')
        KSEED=0
C*      INITIALIZE RAND
        SEED=RAND(-SEED)
        KSEED=KSEED+1
C*      RUN THRU 60 DAYS OF FILE
        DO 10 N=1,60
C*      RANDOMIZE NON-ZERO SIDE 2 STRATEGIES KEEPING COUNT
        K=0
        DO 20 J=1,10
        IF(IIVEC(N,J,2).EQ.0) GO TO 20
        SEED=RAND(SEED)
        KSEED=KSEED+1
        ISEED=SEED*20.+1.
        IIVEC(N,J,2)=ISEED
        K=K+1
     20 CONTINUE
C*      FIND RANDOM NUMBERS FOR EACH FREQUENCY
        SUM=0.
        DO 30 J=1,K
        SEED=RAND(SEED)
        KSEED=KSEED+1
        TFREQ(N,J,2)=SEED
        SUM=SUM+SEED
     30 CONTINUE
```

Figure 9. Red Strategy Program Listing
(Part 1 of 2)

```
C*     NORMALIZE FREQUENCY RANDOM NUMBERS
       DO 40 J=1,K
       TFREQ(N,J,2)=TFREQ(N,J,2)/SUM
   40 CONTINUE
   10 CONTINUE
C*     WRITE RANDOMIZED FILE 12 + NEW SEED + COUNTER
       REWIND 12
       DO 50 IIM=1,60
       DO 50 JJM=1,10
       WRITE(12,2) ((IIVEC(IIM,JJM,KKM),KKM=1,2),
      &(TFREQ(IIM,JJM,KKM),KKM=1,2))
   50 CONTINUE
       WRITE(12,3) SEED
       JSEED=JSEED+1
       WRITE(12,4) JSEED
       WRITE(6,60) JSEED
   60 FORMAT(' RAND WAS USED ',I10,' TIMES TO PRODUCE',
      &' THIS RANDOMIZATION.')
       WRITE(6,65) SEED
   65 FORMAT(' THE FINAL VALUE OF THE SEED IS',E17.10)
       STOP
       END
C*
C*     RAND IS A UNIFORM RANDOM NUMBER GENERATOR,
C*     TAKEN FROM THE HONEYWELL TIME SHARING
C*     LIBRARY.
       FUNCTION RAND(X)
       IF(X) 10,20,20
   20 RN=RHO*RAND
       RN1=AMOD(RN,BN)
       RAND=RN1/BN
       RETURN
   10 RHO=7.0**13
       BN=10.0**10
       RAND=-X
       GO TO 20
       END
```

Figure 9. (Part 2 of 2)